





Electroweak corrections to top-pair production at lepton colliders

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in collaboration with Y. Kurihara, K. Khiem, F. Le Diberder, B. Mecaj, T. Moskalets, R. Poeschl, F. Richard ...

Bi-Weekly meeting of Snowmass topics group EF03 30th July @ home

Top physics at ILC

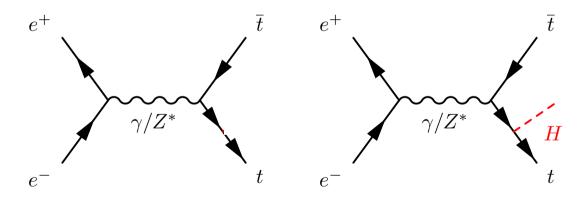


Top being only fermion with mass close to electroweak scale make us think that it has a special role in the physics beyond the SM. Top physics at ILC may open an unique window for discovery of new physics.



Top physics is one of the three pillars of ILC physics*!

- i) top mass measurement at the threshold
- ii) tt-Z/γ anomalous coupling measurement
- iii) tīt-Higgs coupling measurement

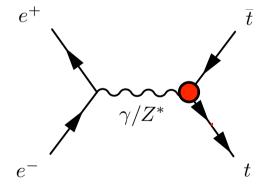


- * ILC baseline energy is 250 GeV.
- * For i), energy upgrade to >350 GeV needed.
- * For ii)&iii) energy upgrade to >500 GeV needed.

The $t\bar{t}$ - Z/γ anomalous coupling

 The tt-z/γ anomalous coupling is one of the important probes of new physics. Many new physics models predict significant deviations from SM.

$$\mathcal{L}_{\text{int}} = \sum_{v=\gamma,Z} g^v \left[V_l^v \bar{t} \gamma^l (F_{1V}^v + F_{1A}^v \gamma_5) t + \frac{i}{2m_t} \partial_\nu V_l \bar{t} \sigma^{l\nu} (F_{2V}^v + F_{2A}^v \gamma_5) t \right]$$

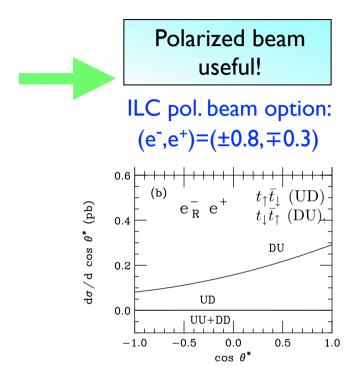


8 form factors to describe the $tt-z/\gamma$ anomalous coupling.

Spin correlation as a tool for discovery

Parke and Shadmi PLB387 ('96) Top production and decays are very different form the other fermions.

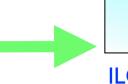
Many (interesting) angular correlations emerge, which can be used to extract various information.



Spin correlation as a tool for discovery

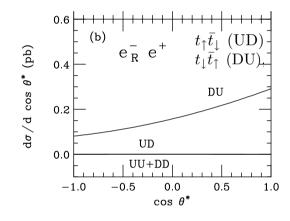
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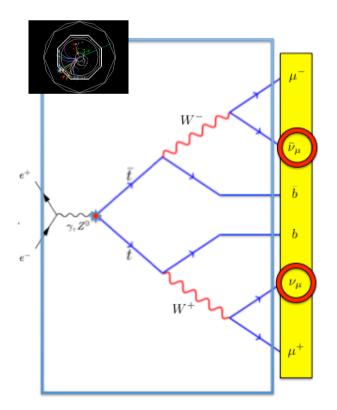
Many (interesting) angular correlations emerge, which can be used to extract various information.



Polarized beam useful!

ILC pol. beam option: $(e^-,e^+)=(\pm 0.8,\mp 0.3)$





Example of fully leptons final state:

Angles for mu+, mu- and b, anti-b (7 angles)

Energies of mu+ and mu
Energies of b and anti-b

A full kinematical information can be reconstructed!

Spin correlation as a tool for discovery

* The recent studies show that these form factor can be measured at precision of the per-mill level. Khiem, et al arXiv:1503:04247

The third measurements presented in table 5 comes from the TESLA TDR, assuming no e^+ polarization, and 300 fb⁻¹ (uncertainties have been rescaled here to 500 fb⁻¹) and are 4 individual fits of the 4 CP-violating form factors: $\mathcal{R}e\ \delta\tilde{F}_{2A}^{\gamma}$, $\mathcal{R}e\ \delta\tilde{F}_{2A}^{Z}$, $\mathcal{I}m\ \delta\tilde{F}_{2A}^{\gamma}$, and $\mathcal{I}m\ \delta\tilde{F}_{2A}^{Z}$:

```
\sigma[\Re \delta \tilde{F}_{2A}^{\gamma}] = 0.005 \quad ; \quad \sigma[\Re \delta \tilde{F}_{2A}^{Z}] = 0.006 \quad ; \quad \sigma[\mathcal{I} \text{m } \delta \tilde{F}_{2A}^{\gamma}] = 0.006 \quad ; \quad \sigma[\mathcal{I} \text{m } \delta \tilde{F}_{2A}^{Z}] = 0.012 \quad (29)
```

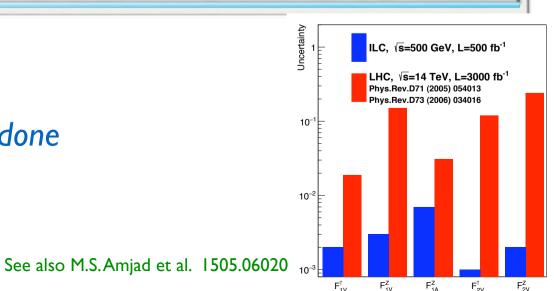
to be compared with the double-leptonic simultaneous fit result (with polarized e^+ Semi-leptonic

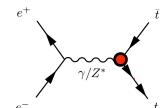
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\sigma[\mathcal{R}e\ \delta \tilde{F}_{2A}^{\gamma}] = 0.007 \quad ; \quad \sigma[\mathcal{R}e\ \delta \tilde{F}_{2A}^{Z}] = 0.012 \quad ; \quad \sigma[\mathcal{I}m\ \delta \tilde{F}_{2A}^{\gamma}] = 0.007 \quad ; \quad \sigma[\mathcal{I}m\ \delta \tilde{F}_{2A}^{Z}] = 0.010 \quad (30)
```

with small correlation coefficients.

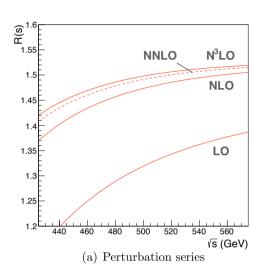
Fully-leptonic

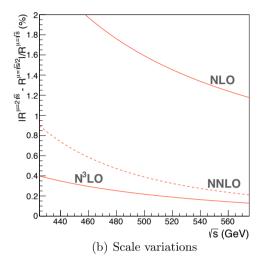
But all these studies have been done at tree level...





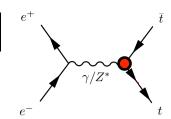
*QCD corrections are known up to N3LO



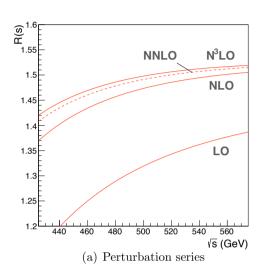


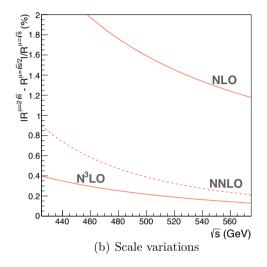
QCD correction (N³LO) is at the per mil level

Kiyo, Maier, Maierhofer, Marquard, NCP B823 ('09)
Bernreuther, Bonciani, Gehrmann, Heinesch,
Leineweber. NPB750 ('06)
Hoang, Mateu, Zebarjad, NPB813 ('09)



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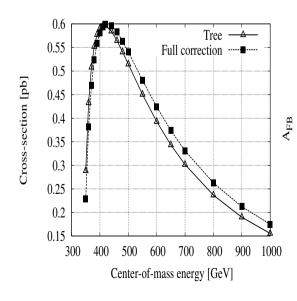


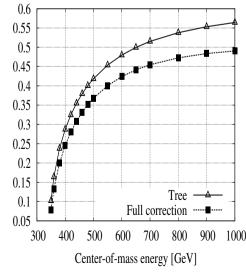


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*Electroweak corrections are known at one-loop level



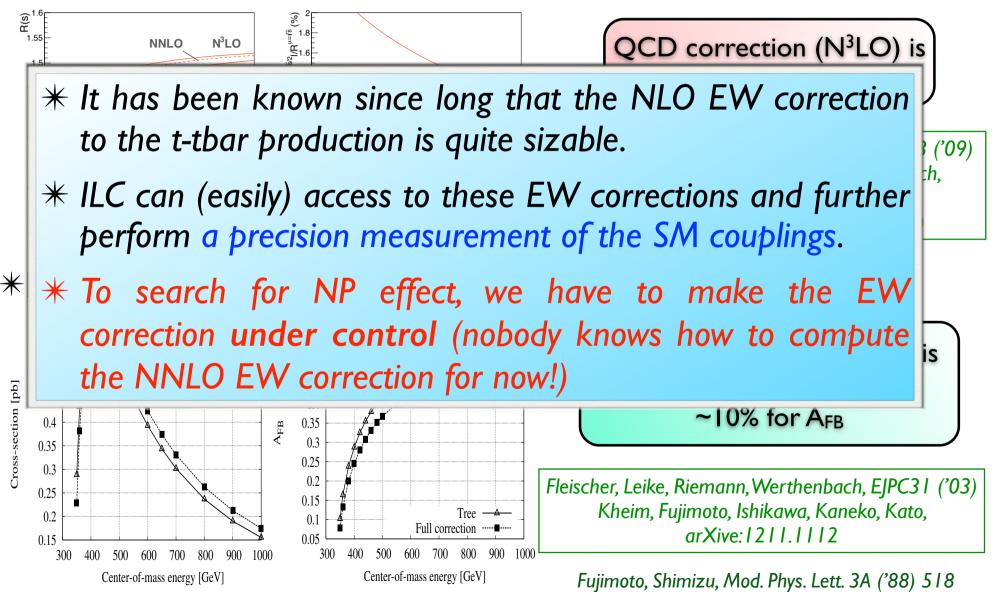


EW correction at one-loop is ~5% for cross section ~10% for A_{FB}

Fleischer, Leike, Riemann, Werthenbach, EJPC3 I ('03) Kheim, Fujimoto, Ishikawa, Kaneko, Kato, arXive: 12 I 1.112

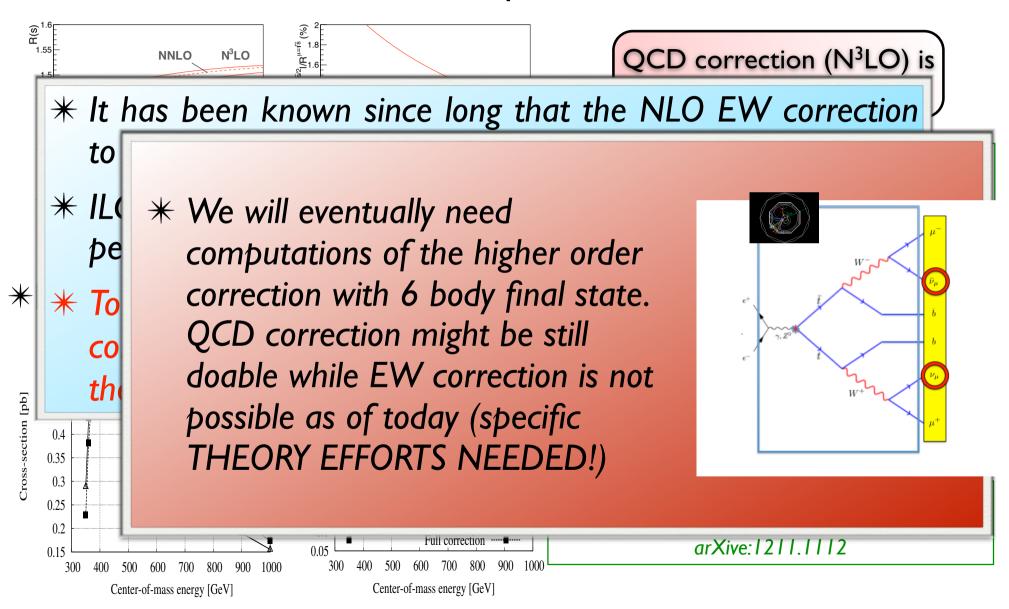
Fujimoto, Shimizu, Mod. Phys. Lett. 3A ('88) 518 Beenkakker, van der Marck, Hollik, Nucl. Phys. B365('91) 24

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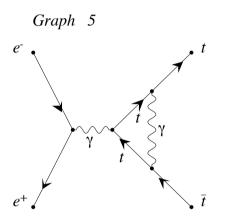
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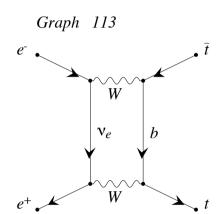


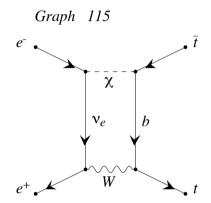
EW@ NLO with GRACE package

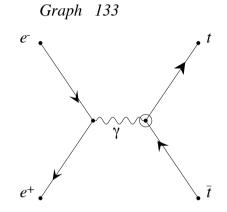
GRACE/GRACE-Loop package: Belanger et al, Phys.Rep.430, 117 Recent progress: Khiem et al, Eur. Phys. J. C 73 ('13), Quach et al, Eur. Phys. J. C78 ('18)

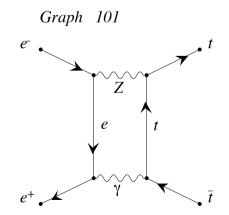
$$|M_0 + \alpha M_{\text{virtual}}|^2 = |M_0|^2 + 2\alpha Re\left(M_0^{\dagger} M_{\text{virtual}}\right) + O(\alpha^2)$$

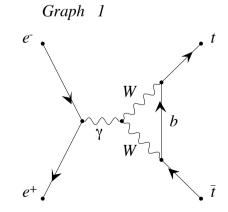








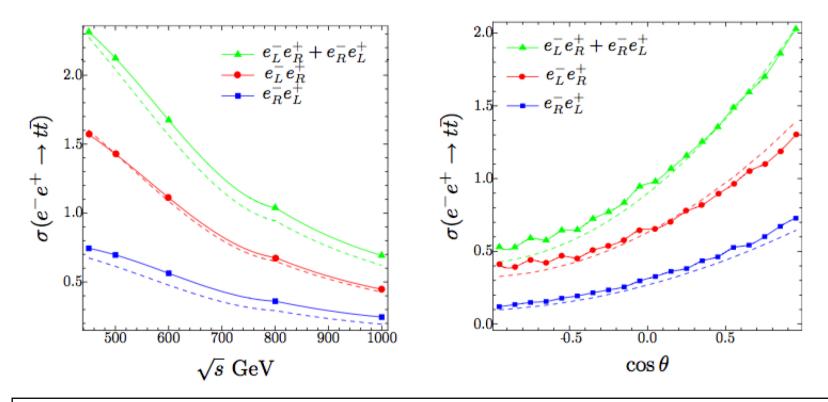




- √ 150 diagrams computed automatically
- √ 'tHoof-Feynman gauge (xi=1)
- √ Goldstone bosons, ghosts etc included
- √ Renormalisation; on-shell scheme
- ✓ IR-div: hard photon cut-off

It turned out that the size of the NLO correction depend strongly on the initial state polarisation.

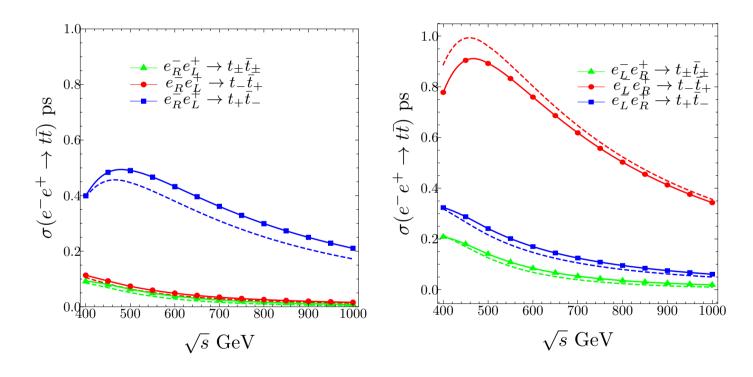
Khiem, E.K. Kurihara, Le Diberder arXiv:1503:04247



- large NLO correction to the cross section: e⁻Le⁺R
- large NLO correction to the forward-backward asymmetry: e-Re+L

It turned out that the size of the NLO correction depend strongly on the final state polarisation as well.

E.K.Y. Kurihara, B. Mecaj, T. Moskalets in preparation



- large NLO correction to the cross section: e⁻Re⁺L→t+tbar₋
- large NLO correction to the forward-backward asymmetry: e⁻Le^{+R}→t₋tbar₊

@500 GeV

Polarization	$\sigma_{NLO}(\mathrm{pb})$	$\sigma_{Tree} \text{ (pb)}$	$\delta_{ m NLO}(\%)$
$e_L^- e_R^+ \to t \bar{t}_+$	0.892	0.960	\sim -7.1
$e_L^- e_R^+ \to t_+ \bar{t}$	0.241	0.216	~ 11.6
$e_L^- e_R^+ \to t \bar{t}$	0.141	0.126	~ 12.0
$e_L^- e_R^+ \to t_+ \bar{t}_+$	0.141	0.126	\sim 12.0
$e_L^- e_R^+ \to (t\bar{t})_{\text{tot.}}$	1.415	1.427	~ -0.9
$e_R^- e_L^+ \to t_+ \bar{t}$	0.490	0.447	~ 9.8
$e_R^- e_L^+ \to t \bar{t}_+$	0.074	0.064	~ 15.2
$e_R^- e_L^+ \to t \bar{t}$	0.063	0.051	~ 23.6
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$e_R^- e_L^+ \to (t\bar{t})_{\rm tot.}$	0.691	0.613	~ 12.6

$$\delta_{
m NLO} \equiv rac{\sigma_{
m NLO} - \sigma_{
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m Tree}}$$

Why NLO correction is large for e^-Re^+L ?

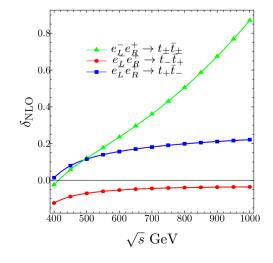
@500 GeV

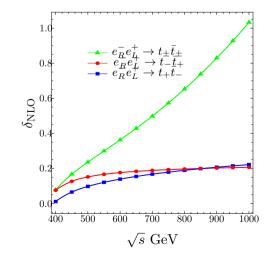
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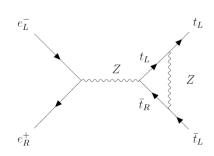
$$\delta_{
m NLO} \equiv rac{\sigma_{
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Forbidden at tree level at massless limit.

But these contributions are very small in any case...







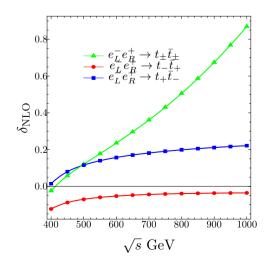
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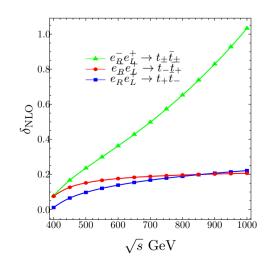
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Why NLO correction is large for e^-Re^+L ?

 \Rightarrow Because t_{\pm} tbar $_{\mp}$ receive the same sign NLO correction.





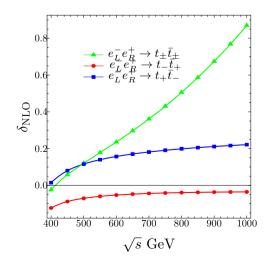
@500 GeV

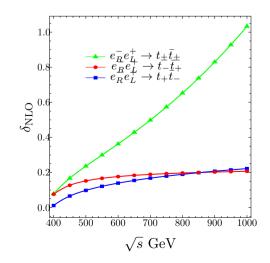
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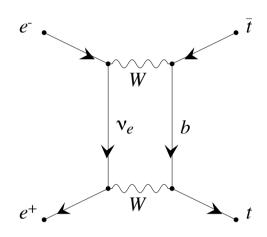
Why NLO correction is large for $e^{-}Re^{+}L$?

- \Rightarrow Because t_{\pm} tbar $_{\mp}$ receive the same sign NLO correction.
- ⇒ Then, why the sign of NLO corruption is opposite for $e^{-}L^{+}R \rightarrow t_{+}tbar_{-}$ and $e^{-}L^{+}R \rightarrow t_{-}tbar_{+}$???





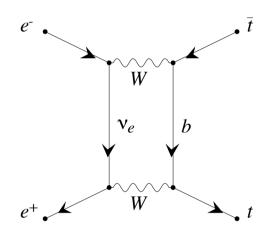
Obvious difference in NLO correction between two initial polarisation is the W box diagram (it is only for e^-Le^+R)!



Polarization	$\sigma_{ m NLO} - \sigma_{ m Tree}(m pb)$	$M_{\text{Tree}} \cdot M_{W-\text{box}} \text{ (pb)}$	The rest of NLO (pb)
$e_L^- e_R^+ \to t \bar{t}_+$	-0.068	-0.161	0.093
$e_L^- e_R^+ \to t_+ \bar{t}$	0.025	-0.006	0.031
$e_L^- e_R^+ \to t \bar{t}$	0.015	-0.013	0.028
$e_L^- e_R^+ \to t_+ \bar{t}_+$	0.015	-0.013	0.028
$e_L^- e_R^+ \to (t\bar{t})_{\rm tot.}$	0.013	-0.194	0.181
$e_R^- e_L^+ \to t_+ \bar{t}$	0.044	0.000	0.044
$e_R^- e_L^+ \to t \bar{t}_+$	0.010	0.000	0.010
$e_R^- e_L^+ \to t \bar{t}$	0.012	0.000	0.012
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@500 GeV

Obvious difference in NLO correction between two initial polarisation is the W box diagram (it is only for e^-Le^+R)!



 \Rightarrow BUT...

is the box diagram gauge independent???

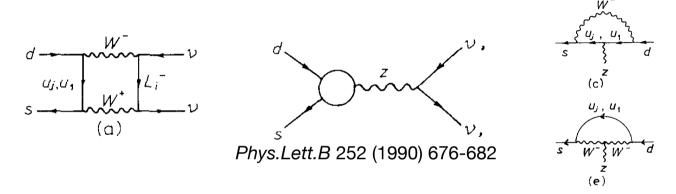
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@500 GeV

Gauge dependence consideration

In the FCNC computation, the gauge dependence cancels between box and vertex correction diagrams...

Inami and Lim, PTP, 65 ('81) 297



For e+ e- -> bbbar, the box, we compute only the vertex correction... but it is because the box is very small near Z pole (also for on-shell Z, it is gauge-invariant)

Lynn and Stuart Phys.Lett.B 252 ('90) 676

Gauge dependence consideration

In the FCNC computation, the gauge dependence cancels between box and vertex correction diagrams...

Inami and Lim, PTP, 65 ('81) 297

 \mathcal{N}_{∞}

* Gauge invariant combination is

 $W/Z/\gamma$ box + W/Z vertex + gauge boson self energy

pole (also for on-shell \angle , it is gauge-invariant)

Lynn and Stuart Phys.Lett.B 252 ('90) 676

Conclusions

- The tt-z/γ anomalous coupling is one of the important probes of new physics. Many new physics models predict significant deviations from SM.
- Experimentally the form factor for tt-z/ γ can be measured at the per-mill level.
- The electroweak NLO correction is known to reach to 5-10%. Thus, it has to be included in the future studies.
- The electroweak NLO correction is sensitive to the initial and final polarisation.
- The polarised ILC will allow us a more detailed study.
- Specific theoretical efforts are needed to make progress on the electroweak correction in the future.

Backup

e+e- → ttbar @ tree level

$$\mathcal{L}_{\text{int}} = \sum_{v=\gamma,Z} g^v \left[V_l^v \bar{t} \gamma^l (F_{1V}^v + F_{1A}^v \gamma_5) t + \frac{i}{2m_t} \partial_\nu V_l \bar{t} \sigma^{l\nu} (F_{2V}^v + F_{2A}^v \gamma_5) t \right]$$

$$\mathcal{M}(e_L \bar{e}_R \to t_L \bar{t}_R)^{\gamma/Z} = c_L^{\gamma/Z} [F_{1V}^{\gamma/Z} - \beta F_{1A}^{\gamma/Z} + F_{2V}^{\gamma/Z}] (1 + \cos \theta) e^{-i\phi}$$

$$\mathcal{M}(e_L \bar{e}_R \to t_R \bar{t}_L)^{\gamma/Z} = c_L^{\gamma/Z} [F_{1V}^{\gamma/Z} + \beta F_{1A}^{\gamma/Z} + F_{2V}^{\gamma/Z}] (1 - \cos \theta) e^{-i\phi}$$

$$\mathcal{M}(e_L \bar{e}_R \to t_L \bar{t}_L)^{\gamma/Z} = c_L^{\gamma/Z} \gamma^{-1} [F_{1V}^{\gamma/Z} + \gamma^2 (F_{2V}^{\gamma/Z} + \beta F_{2A}^{\gamma/Z})] \sin \theta e^{-i\phi}$$

$$\mathcal{M}(e_L \bar{e}_R \to t_R \bar{t}_R)^{\gamma/Z} = c_L^{\gamma/Z} \gamma^{-1} [F_{1V}^{\gamma/Z} + \gamma^2 (F_{2V}^{\gamma/Z} - \beta F_{2A}^{\gamma/Z})] \sin \theta e^{-i\phi}$$

$$\mathcal{M}(e_R \bar{e}_L \to t_L \bar{t}_R)^{\gamma/Z} = -c_R^{\gamma/Z} [F_{1V}^{\gamma/Z} - \beta F_{1A}^{\gamma/Z} + F_{2V}^{\gamma/Z}] (1 - \cos \theta) e^{i\phi}$$

$$\mathcal{M}(e_R \bar{e}_L \to t_R \bar{t}_L)^{\gamma/Z} = -c_R^{\gamma/Z} [F_{1V}^{\gamma/Z} + \beta F_{1A}^{\gamma/Z} + F_{2V}^{\gamma/Z}] (1 + \cos \theta) e^{i\phi}$$

$$\mathcal{M}(e_R \bar{e}_L \to t_L \bar{t}_L)^{\gamma/Z} = c_R^{\gamma/Z} \gamma^{-1} [F_{1V}^{\gamma/Z} + \gamma^2 (F_{2V}^{\gamma/Z} + \beta F_{2A}^{\gamma/Z})] \sin \theta e^{i\phi}$$

$$\mathcal{M}(e_R \bar{e}_L \to t_R \bar{t}_R)^{\gamma/Z} = c_R^{\gamma/Z} \gamma^{-1} [F_{1V}^{\gamma/Z} + \gamma^2 (F_{2V}^{\gamma/Z} - \beta F_{2A}^{\gamma/Z})] \sin \theta e^{i\phi}$$

$$\mathcal{M}(e_R \bar{e}_L \to t_R \bar{t}_R)^{\gamma/Z} = c_R^{\gamma/Z} \gamma^{-1} [F_{1V}^{\gamma/Z} + \gamma^2 (F_{2V}^{\gamma/Z} - \beta F_{2A}^{\gamma/Z})] \sin \theta e^{i\phi}$$

where $\beta^2 = 1 - 4m_t^2/s$, $\gamma = \sqrt{s}/(2m_t)$ and the overall factors $c_{L/R}^{\gamma/Z}$ are:

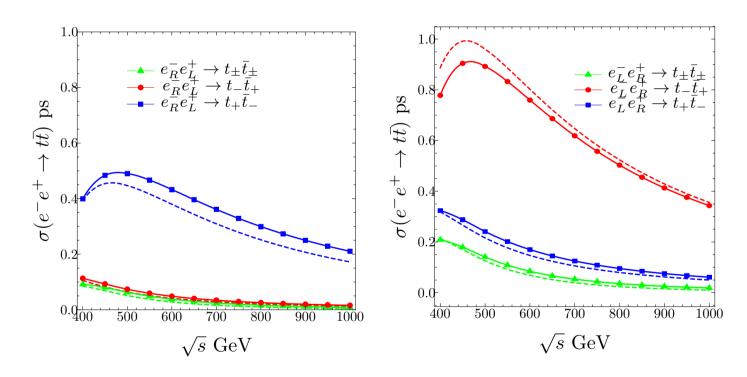
$$c_L^{\gamma} = -1, \quad c_R^{\gamma} = -1, \quad c_L^{Z} = \left(\frac{-1/2 + s_w^2}{s_w c_s}\right) \left(\frac{s}{s - m_Z^2}\right), \quad c_R^{Z} = \left(\frac{s_w^2}{s_w c_s}\right) \left(\frac{s}{s - m_Z^2}\right)$$

where $s_w = \sin \theta_w$ and $c_w = \cos \theta_W$, with θ_W being the weak mixing angle.

$$F_{1V}^{\gamma,SM} = -\frac{2}{3}, F_{1A}^{\gamma,SM} = 0, F_{1V}^{Z,SM} = -\frac{1}{4s_w c_w} \left(1 - \frac{8}{3} s_w^2 \right), F_{1A}^{Z,SM} = \frac{1}{4s_w c_w}, \quad F_{2V}^{\gamma} = Q_t(g-2)/2$$

It turned out that the size of the NLO correction depend strongly on the final state polarisation as well.

E.K.Y. Kurihara, B. Mecaj, T. Moskalets in preparation



- large NLO correction to the cross section: e⁻Re⁺L→t+tbar₋
- large NLO correction to the forward-backward asymmetry: e⁻Le^{+R}→t₋tbar₊